

**B.M.C Durfee High School**  
**AP Calculus BC**  
**Summer Assignment 2010**

***Congratulations! You made it to BC Calculus!***

In order to complete the curriculum before the AP Exam in May, it is necessary to do some preparatory work this summer. This review packet will help you to focus on the mathematical skills and content you will need to use in solving Calculus problems. These problems deal with skills and content that you studied in Pre-Calculus and Calculus AB. Use your Pre-Calculus and Calculus notes to help you solve the review problems. There are many websites on the internet that can also be helpful in completing these problems.

**You are responsible for completing this summer assignment by the first day of the new school year.** The assignment will be collected and counted as a test grade for the 1<sup>st</sup> term of Calculus BC. In order to receive full credit, complete work on separate paper must be shown to justify your answers and graphs must be carefully drawn and labeled. If a calculator has been used, then you must set up what you entered into the calculator and what the calculator produced for you on your paper.

At this level, doing homework is more than just getting the problems done. The problems should be a learning experience. It is strongly recommended that you do a few problems each day throughout the summer. **Do not leave the entire assignment for the night before school starts.** If you are having trouble with some of the problems, please contact me at [jachadinha@fallriverschools.org](mailto:jachadinha@fallriverschools.org). I will be checking my e-mail on a daily basis throughout the summer. I will also be available on Wednesdays (starting July 7<sup>th</sup>) from 9:00 am to 11:00 am at the IMC for help with the assignment.

**Best of luck with the assignment! I look forward to seeing you in September!**

Mrs. Achadinha

- Find an equation for the line that contains the points (2, -3) and (6, 9).
- Find the value of  $y$  for which the line through  $A$  and  $B$  has the given slope  $m$ :  $A(-2, 3)$ ,  $B(4, y)$ ,  

$$m = -\frac{2}{3}.$$
- Find an equation for the line that contains the coordinate (5, 1) and is perpendicular to the line  $6x - 3y = 2$ .
- For what value of  $k$  are the two lines  $2x + ky = 3$  and  $x + y = 1$ 
  - Parallel?
  - Perpendicular?

**For questions 5-10, let  $f(x) = \sqrt{x-3}$  and  $g(x) = x^2 + 5$ .**

- $(f + g)(-3)$
- $(g - f)(6)$
- $g^{-1}(x)$
- $(g \circ f)(x)$
- $\frac{1}{f(x)}$
- $(f \circ g)(3)$
- Algebraically find the inverse of  $y = \frac{3}{x-2} - 1$ .
- If  $f(x) = x^3 - 1$ , find  $f^{-1}$  and verify that  $f(f^{-1}(x)) = f^{-1}(f(x)) = x$ .

**For questions 13-20, simplify each expression completely.**

- $\frac{\sqrt{x}}{x}$
- $e^{\ln 3}$
- $\ln 1$
- $\ln e^7$
- $\log_{\frac{1}{2}} 8$
- $e^{3 \ln x}$

$$19. \frac{4xy^{-2}}{12x^{\frac{-1}{3}}y^{-5}}$$

$$20. 27^{\frac{2}{3}}$$

21. Rewrite  $\frac{1}{2}\ln(x-3) + \ln(x+2) - 6\ln x$  as a single logarithmic expression.

22. Solve for  $t$ :  $(1.045)^t = 2$

23. Solve for  $x$ :  $\log_5 x + \log_5(x-4) = 1$

24. Solve for  $x$ :  $27^{2x} = 9^{x-3}$

25. Solve for  $x$ :  $\ln(3x)^2 = 16$

26. Evaluate  $\log_2 5$  to the nearest thousandth.

27. The number of elk after  $t$  years in a state park is modeled by the function  $P(t) = \frac{1216}{1 + 75e^{-0.03t}}$

a) What was the initial population of elk?

b) When will the number of elk be 750?

c) What is the maximum number of elk possible in the park?

28. Arturo invests \$2700 in a savings account that pays 9% interest, compounded quarterly. If there are no other transactions, when will his balance reach \$4550?

29. Without using a calculator, find the exact value of  $\cos^{-1}\left(\cos\left(\frac{17\pi}{5}\right)\right)$ . Justify your answer.

**For questions 30-35, find the exact values of each trigonometric function.**

30.  $\sin\left(\frac{7\pi}{6}\right)$

31.  $\csc(60^\circ)$

32.  $\cos(20^\circ)$

33.  $\sec\left(-\frac{2\pi}{3}\right)$

34.  $\tan\left(\frac{\pi}{2}\right)$

35.  $\cot(-135^\circ)$

36. Simplify  $(\csc(x) - \tan(x))\sin(x)\cos(x)$

37. List the three Pythagorean Identities.

38. List the double angle formulas

a)  $\sin 2x =$

b)  $\cos 2x =$

39. List the sum and difference formulas.

a)  $\cos(\alpha \pm \beta) =$

b)  $\sin(\alpha \pm \beta) =$

40. Prove that  $\frac{\sin \theta}{1 - \cos \theta} + \frac{1 - \cos \theta}{\sin \theta} = 2 \csc \theta$

41. Prove that  $(\sin x + \cos x)^2 = 1 + \sin 2x$

42. Determine the range of  $f(x) = 13 - 20x - x^2 - 3x^4$ . Also, find the maximum and minimum values of  $f(x)$ , and state where those values occur.

43. Find the equation for the parabola whose vertex is  $(2, -5)$  and passes through  $(4, 7)$ . Express your answer in standard form for a quadratic.
44. Consider the circle of the radius 5 centered at  $(0, 0)$ . Find an equation of the line tangent to the circle at the point  $(3, 4)$  in slope-intercept form.
45. Determine all points of intersection for  $y = x^2 + 3x - 4$  and  $y = 5x + 11$ .
46. Find the points of intersection in the graphs of  $y = x - 1$  and  $y^2 = 2x + 6$ .
47. Use a graphing calculator to approximate all of the function's real zeros. Round your results to 3 decimal places.  $f(x) = 3x^6 - 5x^5 - 4x^3 + x^2 + x + 1$
48. Algebraically determine whether the function is even, odd, or neither:  $y = \frac{x^3}{x^2 - 1}$
49. Determine whether the function is even, odd, or neither:  $f(x) = 1 + \sin x$
50. Graph the piecewise function on the attached graph paper.

$$f(x) = \begin{cases} x^2 & x < 0 \\ x + 2 & 0 \leq x \leq 3 \\ 4 & x > 3 \end{cases}$$

51. For the function below, give the zeros (if non exist, write *none*), domain, range, VA's, HA's, and/or points of discontinuity (holes-as ordered pairs)if any exist. Also sketch the function's graph.

$$f(x) = \frac{x + 3}{2x^2 + 5x - 3}$$

For questions 52-55, graph each function on the attached graph paper. Give its domain and range.

52.  $y = -e^{-x}$

53.  $y = |x+3| - 2$

54.  $y = 3 - 2 \sin x$

55.  $y = 1 + \sqrt{x+2}$

56. Eliminate the parameter and write a rectangular equation for  $\begin{matrix} x = t^2 + 3 \\ y = 2t \end{matrix}$

57. Expand and simplify.

a)  $\sum_{n=0}^4 \frac{n^2}{2}$

b)  $\sum_{n=1}^3 \frac{1}{n^3}$

58. Find the  $\lim_{x \rightarrow 1} \frac{x-1}{x^2-1}$

59. Find the  $\lim_{x \rightarrow 5} (2x^2 - 3x + 4)$

60. Find the  $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

61. Find the  $\lim_{x \rightarrow -3} \frac{x^2 + x - 6}{x^2 - 9}$

62. Find the  $\lim_{x \rightarrow 5^+} \ln(x-5)$

63. Find the  $\lim_{t \rightarrow -\infty} \frac{6t^2 + 5t}{(1-t)(2t-3)}$

64. Find the  $\lim_{x \rightarrow \infty} \cos x$

65. Find the  $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x}$

66. Find the  $\lim_{x \rightarrow \infty} \frac{2x^2}{5x^2 - 9x - 2}$

67. Find the  $\lim_{x \rightarrow \infty} \frac{x^2 + x}{3 - x}$

68. Find the  $\lim_{x \rightarrow 3^-} f(x)$ , where  $f(x) = \begin{cases} \frac{x+2}{2}, & x \leq 3 \\ \frac{12-2x}{3}, & x > 3 \end{cases}$

69. Find the slope of the tangent line to the graph of  $g(x) = x^2 - 4$  at the point  $(1, -3)$ .

70. Find an equation of the tangent line to the hyperbola  $y = \frac{3}{x}$  at the point  $(3, 1)$ .

71. Find an equation of the tangent line to the graph of  $f(x) = x^2 - 8x + 9$  at the point  $(3, -6)$ .

**For questions 72-74, find the x-values (if any) at which  $f$  is not continuous. Which of the discontinuities are removable.**

72.  $f(x) = \frac{x}{x^2 - x}$

73.  $f(x) = 3x - \cos x$

74.  $f(x) = \frac{x+2}{x^2 - 3x - 10}$

**For questions 75-94, find the derivative of each given function.**

75.  $f(x) = x^{-\frac{2}{5}}$

76.  $f(x) = x^3 - x^2 + 2x$ .

77.  $y = 5e^x + 3$

78.  $f(x) = \sqrt[3]{x}$

79.  $g(x) = \frac{\sqrt{10}}{x^7}$

80.  $f(x) = \frac{x^2 + 4x + 3}{\sqrt{x}}$

81.  $f(x) = x^2 e^x$

82.  $f(x) = \frac{e^x}{x^2}$

83.  $f(x) = \frac{x}{(8-3x)}$

84.  $\tan \sqrt{1-x}$

85.  $y = \sin x + \cos x$

86.  $y = x^2 \cos x$

87.  $f(x) = \cos x - 2 \tan x$

88.  $f(x) = 2 \cot x - \sqrt{x} \sec x$

89.  $g(x) = (x^3 - 1)^{100}$

90.  $y = \sqrt{x^2 - 7x}$

91.  $y = (2x+1)^5 (x^3 - x + 1)^4$

92.  $y = \ln(x^3 + 1)$

93. Find  $y'$  of  $y = \log_{10} x$ .

94. If  $f(x) = x^2 - 2e^x$ , find the value of  $f'(1)$

95. If  $f(x) = 2x^2 - x^3$ , find  $f'(x)$ ,  $f''(x)$ , and  $f'''(x)$

**For questions 96-97, find  $\frac{dy}{dx}$  by implicit differentiation.**

96.  $x^2 + y^2 = 25$

97.  $x^2 - xy + y^3 = 8$

98. If a billiard ball is dropped from a height of 100 feet, its height  $s$  at time  $t$  is given by the position function:  $s = -16t^2 + 100$  where  $s$  is measured in feet and  $t$  is measured in seconds. Find the average velocity of each the following time intervals.
- a)  $[1, 2]$                       b)  $[1, 1.5]$                       c)  $[1, 1.1]$
99. Suppose that a ball is dropped from the upper observation deck of the CN Tower, 450 m above the ground.
- a) What is the velocity of the ball after 5 seconds?
- b) How fast is the ball traveling when it hits the ground?
100. A particle moves according to a law of motion  $s = f(t) = t^3 - 12t^2 + 36t$ ,  $t \geq 0$ , where  $t$  is measured in seconds and  $s$  in meters.
- a) Find the velocity at time  $t$ .
- b) What is the velocity after 3 s?
- c) When is the particle at rest?
- d) Find the total distance traveled during the first 8 s.
- e) Find the acceleration at time  $t$ .
- f) Find the acceleration after 3 s.